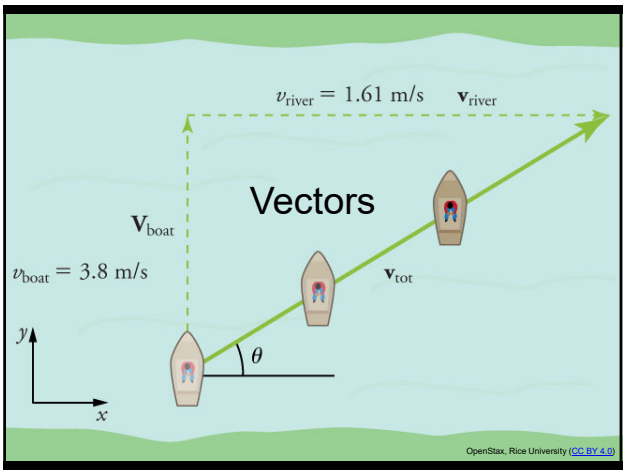
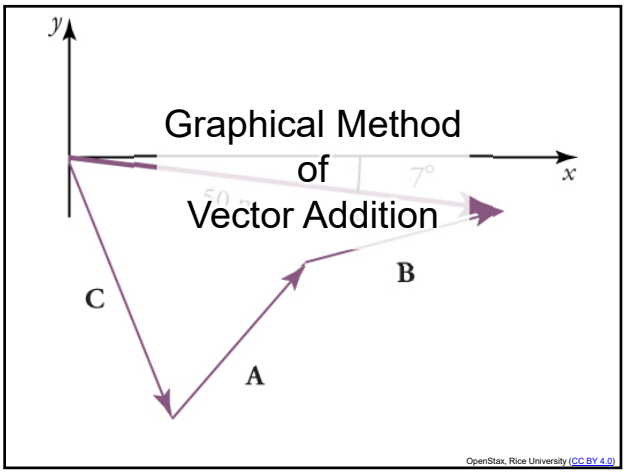
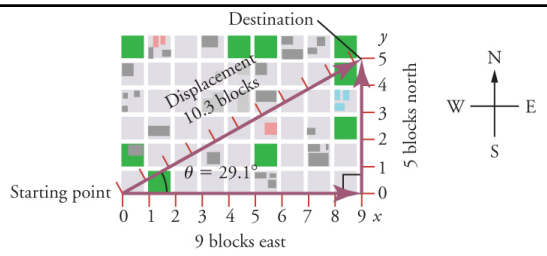
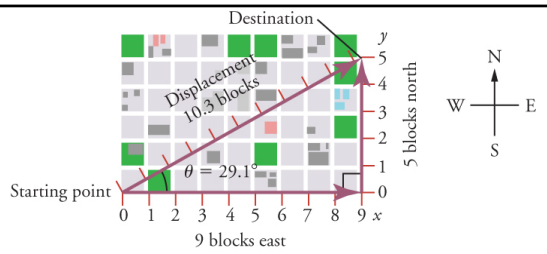


- Recall that a vector is a quantity that has magnitude and direction.
  - displacement, velocity, acceleration, and force
- In one-dimensional or straight-line motion, the direction of a vector can be given simply by a plus or minus sign.
- In two dimensions, a vector describes motion in two perpendicular directions, such as vertical and horizontal.
- For vertical and horizontal motion, each vector is made up of vertical and horizontal components.



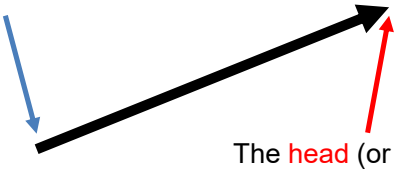


- The figure shows a graphical representation of a vector; the total displacement for a person walking in a city. The person first walks nine blocks east and then five blocks north.



- Her total displacement does not match her path to her final destination. The displacement simply connects her starting point with her ending point using a straight line, which is the shortest distance.

The tail of the vector is the starting point of the vector.



The head (or tip) of a vector is the pointed end of the arrow.

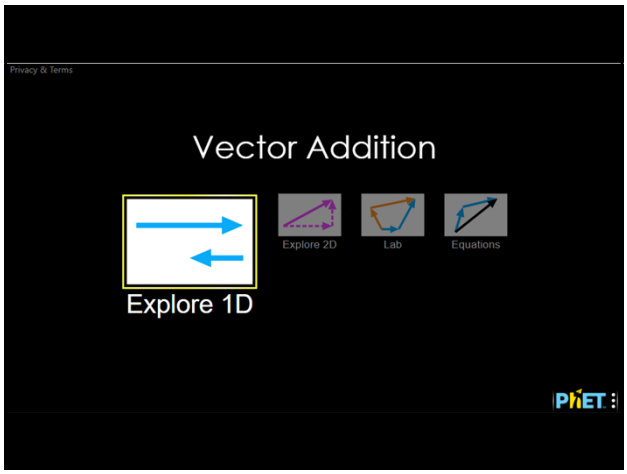
Steps

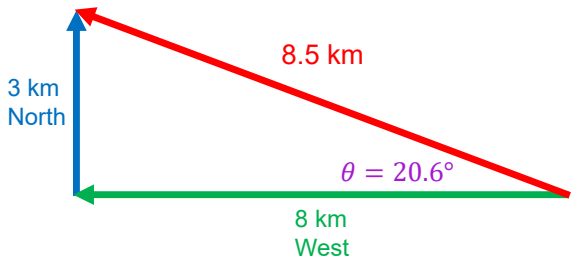
1. Using a ruler and protractor, draw an arrow to represent the first vector.
2. Draw an arrow to represent the second vector. Place the tail of the second vector at the head of the first vector. (For subtraction, reverse the vector.)
3. If there are more than two vectors, continue to add the vectors head-to-tail as described in step 2.

4. Draw an arrow from the tail of the first vector to the head of the last vector. This is the resultant, or the sum, of the vectors.
5. To find the magnitude of the resultant, measure its length with a ruler.
6. To find the direction of the resultant, use a protractor to measure the angle it makes with the reference direction.

Example

- A boy rides his bicycle 8 km West and then 3 km North. Calculate the displacement of the boy.





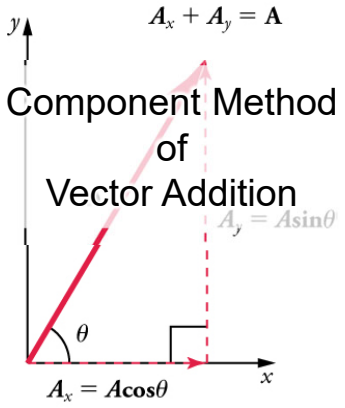
The displacement is 8.5 km 20.6° North of West.

Example 2

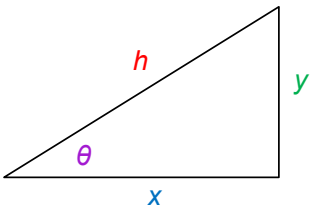
- A boat attempts to travel straight across a river at a speed of 3.8 m/s. The river current flows at a speed of 6.1 m/s to the right. What is the total velocity and direction of the boat?

7.2 m/s 32.0° with respect to the shore.

Component Method of Vector Addition



Trigonometric Ratios



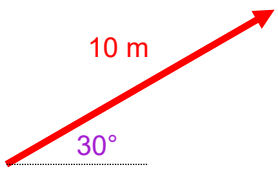
$$\sin\theta = \frac{y}{h}$$

$$\cos\theta = \frac{x}{h}$$

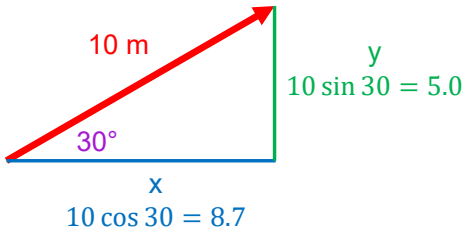
$$\tan\theta = \frac{y}{x}$$

Vector Components

- Consider a vector 10 m 30° north of east.



- The components are calculated using trigonometry.
  - x-component: $10 \cos 30 = 8.7$
  - y-component: $10 \sin 30 = 5.0$



y
$10 \sin 30 = 5.0$

x
$10 \cos 30 = 8.7$

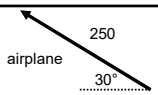
Steps

1. Calculate the components of each of the vectors.
2. Add all the x-components together.
3. Add all the y-components together.
4. Create a triangle using the sum of the x-components, the sum of the y-components, and a line joining them (the hypotenuse).

5. Calculate the hypotenuse using the Pythagorean theorem.
6. Calculate the angle between the x-component and the hypotenuse using trigonometry.

Example 1

- An airplane has a velocity of 250 m/s 30° north of west. The wind has a velocity of 12 m/s 25° north of east. Calculate the resulting speed of the airplane (known as ground speed).



x

$-250 \cos 30 = -216.5$

y

$250 \sin 30 = 125$

x

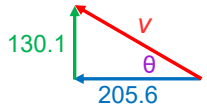
$12 \cos 25 = 10.9$

y

$12 \sin 25 = 5.1$

$x = -216.5 + 10.9 = -205.6$

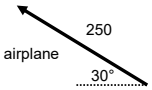
$y = 125 + 5.1 = 130.1$

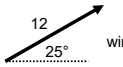


$v = \sqrt{(205.6)^2 + (130.1)^2} = 243.3 \text{ m/s}$

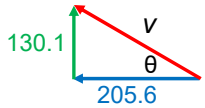
$\theta = \tan^{-1}\left(\frac{130.1}{205.6}\right) = 32.3°$

The resultant velocity is 243.3 m/s 32.3° North of West.





	x-component	y-component
Airplane	$-250 \cos 30 = -216.5$	$250 \sin 30 = 125$
Wind	$12 \cos 25 = 10.9$	$12 \sin 25 = 5.1$
Sum	-205.6	130.1



$v = \sqrt{(205.6)^2 + (130.1)^2}$

$v = 243.3 \text{ m/s}$

$\theta = \tan^{-1}\frac{130.1}{205.6}$

$\theta = 32.3°$

The resultant velocity is 243.3 m/s 32.3° North of West.

Example 2

- A girl first walks 53.0 m in a direction 20.0° north of east then walks 34.0 m in a direction 63.0° north of east. Calculate the displacement.

81.2 m 36.6° north of east

53.0 m, 20°

x

$53 \cos 20 = 49.8$

y

$53 \sin 20 = 18.1$

34.0 m, 63°

x

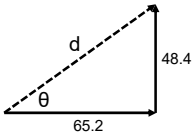
$34 \cos 63 = 15.4$

y

$34 \sin 63 = 30.3$

$x = 49.8 + 15.4 = 65.2$

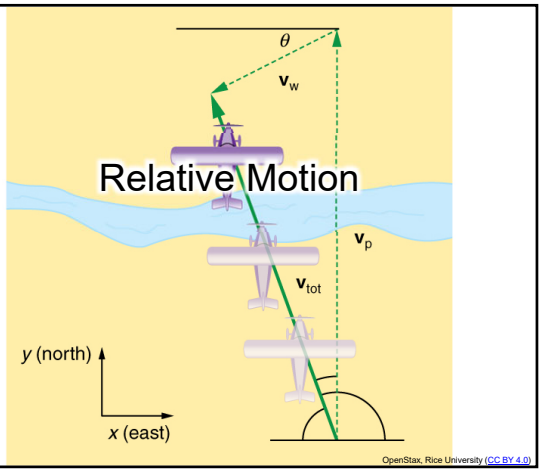
$y = 18.1 + 30.3 = 48.4$



$d = \sqrt{(65.2)^2 + (48.4)^2} = 81.2$

$\theta = \tan^{-1}\left(\frac{48.4}{65.2}\right) = 36.6°$

The displacement is 81.2 m 36.6° North of East.



What Is Relative Velocity?

https://youtu.be/xBS01yzd0yI

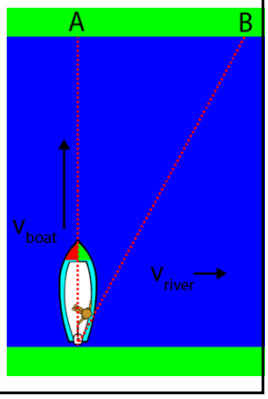
- If a person rows a boat across a rapidly flowing river and tries to head directly for the other shore, the boat instead moves diagonally relative to the shore, because the river carries the boat downstream.
- The boat has a velocity relative to a river and the river has a velocity relative to an observer on solid ground.
- The velocity of the boat relative to the observer is the sum of these velocity vectors.

Example

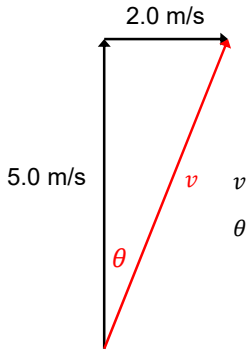
A boat with a velocity of 5.0 m/s is crossing a 50.0 m wide river with a current of 2.0 m/s towards the east.

a) What is the velocity of the boat relative to the shore?

b) What is the distance from A to B?



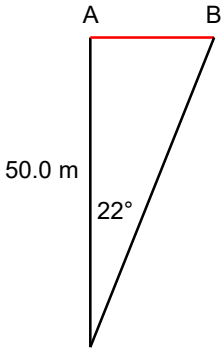
a) Velocity relative to the shore



$v = \sqrt{5^2 + 2^2} = 5.4 \text{ m/s}$

$\theta = \tan^{-1}\left(\frac{2}{5}\right) = 22°$ east of north

b) Distance from A to B



$d = 50 \tan 22 = 20.2 \text{ m}$